Text S2. Calculation of incidence rate ratios and 95% confidence intervals


An effect estimate and 95% confidence interval was not reported in this study. However, the data necessary to calculate a rate ratio has been provided in the supplementary appendix. First, we must calculate the point estimate:

\[ \hat{IR} = \frac{A_1/T_1}{A_0/T_0} \]

A1 = 17 = tuberculosis cases in immediate ART arm
T1 = 1661.9 = person-years at risk of a clinical event in immediate ART arm

A0 = 33 = tuberculosis cases in deferred ART arm
T0 = 1641.8 = person-time at risk of a clinical event in deferred ART arm

\[ IR = \frac{(17/1661.9)}{(33/1641.8)} = 0.5089 \]

Next, we must calculate the standard deviation of the log rate ratio:

\[ SD[\ln(\hat{IR})] = \left(\frac{1}{A_1} + \frac{1}{A_0}\right)^{0.5} \]

\[ SD[\ln(IR)] = (1/17 + 1/33)^{0.5} = 0.2985 \]

Finally, we can calculate the lower and upper limits of the rate ratio:

\[ IR, \hat{IR} = \exp[\ln(\hat{IR}) \pm Z_{\gamma}SD[\ln(\hat{IR})]] \]

\[ = \exp[\ln(0.5089) \pm 1.96(0.2985)] = 0.2835, 0.9136 \]

Therefore the rate ratio and its 95% confidence interval is: 0.51 (0.28 to 0.91).
An effect estimate and 95% confidence interval was not reported in participants with baseline CD4 counts < 200 cells/µL. However, the data necessary to calculate a rate ratio has been provided. In order to calculate the tuberculosis rate ratio for people with CD4 counts < 200 cells/µL we must use the incidence rates and their 95% confidence intervals.

The incidence rate in people who started ART with baseline CD4 counts < 200 cells/µL was 0.60 cases / 100 person-years of observation (95% CI, 0.15 to 2.37). Given

$$\hat{SD}[\ln(\hat{IR})] = \frac{1}{A^{1/2}}$$

we can solve for the (1/A^0.5), i.e. the standard deviation of the log incidence rate, using the lower 95% confidence interval:

$$\hat{IR}, \bar{IR} = \exp[\ln(\hat{IR}) \pm Z_y(1/A^{1/2})]$$

$$\ln(95\%\; IR \; Lower \; Limit, \; LL) = \ln(\hat{IR}) - Z_y \; SD[\ln(\hat{IR})]$$

$$\frac{(\ln(\hat{LL}) - \ln(\hat{IR}))}{-Z_y} = SD[\ln(\hat{IR})]$$

$$SD[\ln(\hat{IR})] = \frac{(\ln(0.15) - \ln(0.60))}{-1.96} = 0.7073$$

Given SD[ln(IR)] we can now calculate A1, or the number of events in the stratum on ART:

$$\hat{SD}[\ln(\hat{IR})] = \frac{1}{A^{1/2}}$$

$$0.7073 = 1/((A1^{0.5})$$

$$1 = 0.7073*(A1^{0.5})$$

$$1 / 0.7073 = (A1^{0.5})$$

$$1.4138 = (A1^{0.5})$$

$$1.9989 = A1$$

A1~2 cases of tuberculosis
The incidence rate in people off ART with baseline CD4 counts < 200 cells/µL was 5.47 cases / 100 person-years of observation (95% CI, 2.73 to 10.94). Given

\[ \hat{SD}[\ln(\hat{IR})] = \frac{1}{A^{0.5}} \]

we can solve for the \((1/A^{0.5})\), i.e. the standard deviation of the log incidence rate, using the lower 95% confidence interval:

\[ \ln(95\% \text{ IR Lower Limit, LL}) = \ln(\text{IR}) - Z_y \, SD[\ln(\text{IR})] \]

\[ (\ln(\text{LL}) - \ln(\text{IR})) / -Z_y = SD[\ln(\text{IR})] \]

\[ SD[\ln(\text{IR})] = (\ln(2.73)-\ln(5.47)) / -1.96 = 0.3546 \]

Given \(SD[\ln(\text{IR})]\) we can now calculate \(A_0\), or the number of events in the stratum off ART:

\[ 0.3546 = 1/(A_0^{0.5}) \]
\[ 1 = 0.3546(A_0^{0.5}) \]
\[ 1 / 0.3546 = (A_0^{0.5}) \]
\[ 2.8202 = (A_0^{0.5}) \]
\[ 7.9537 = A_0 \]

\(A_0 \sim 8\) cases of tuberculosis

Since we have calculated the number of cases in both study arms, we can now calculate the rate ratio and its 95% confidence interval. First, we must calculate the point estimate:

\[ \hat{IR} = \frac{A_1/T_1}{A_0/T_0} \]

\[ IR = (0.60/100) / (5.47/100) = 0.1097 \]

Next we must calculate the standard deviation of the log rate ratio:

\[ \hat{SD}[\ln(\hat{IR})] = \left( \frac{1}{A_1} + \frac{1}{A_0} \right)^{0.5} \]

\[ SD[\ln(\text{IR})] = (1/2 + 1/8)^{0.5} = 0.7912 \]

Finally, we can calculate the 95% limits of the rate ratio:

\[ \hat{IR}, \hat{IR} = \exp[\ln(\hat{IR}) \pm Z_y \, \hat{SD}[\ln(\hat{IR})]] \]
\[ = \exp[\ln(0.1097) \pm 1.96(0.7912)] = 0.0233, 0.5172 \]

Therefore the rate ratio and its 95% confidence interval is: 0.11 (0.02 to 0.52).