

Assessing framing assumptions in quantitative health impact assessments: A housing intervention example

Appendix A

A.1 CONSTRUCTING THE CAUSAL DIAGRAMS FROM INDIVIDUAL STUDIES IDENTIFIED FROM THE LITERATURE

A causal diagram is constructed based on each published study (k) identified in the literature search. For each identified study k , causal weights ($a_{ij}^{(k)}$) are derived from the reported measures of effects to describe the “strength of the causal associations” between the variables (nodes). The causal weight of an association is defined as the natural logarithm of the odds ratio (OR), if it is reported in a study between a pair of nodes i.e.

$$a_{ij}^{(k)} = \ln (OR_{ij}^{(k)}) \quad [1]$$

$a_{ij}^{(k)}$ represents the measure of effect (or causal weight) between two nodes i and j . Associations expressed in other measures of effect, such as correlation coefficient or standardized mean difference, can be converted into odds ratio ($OR_{ij}^{(k)}$) (Lipsey and Wilson 2001) to obtain the corresponding causal weight ($a_{ij}^{(k)}$). If a measure of effect is reported as a percentage changes it is turned into a causal weights by expressing it in a decimal form (e.g. 50% = 0.50). If the measure of effects are not provided in a study, the causal weights are set to 0 for that pair of variables and for that study.

A.2 REPRESENTING MATHEMATICALLY THE COMBINED CAUSAL DIAGRAM INTO A SYSTEM MATRIX

Each study k can be used to construct a causal diagram. A causal diagram can be represented mathematically by a $N \times N$ “connection matrix” (A) with N nodes such that the elements of the matrix A are given by

$$a_{ij}^{(k)} = \left\{ \begin{array}{l} \pm \log (OR_{ij}^{(k)}) \\ 0 \end{array} \right\} \quad [2]$$

Each $(i j)^{th}$ element of the connection matrix $A_k = \{a_{ij}^{(k)}, i = 1..N, j = 1..N\}$ represents the measure of effect (or causal weight) between two nodes i and j . The causal weights are algebraic numbers which can be positive or negative. If $a_{ij}^{(k)} = 0$, it means that the nodes i and j are not connected. The connection matrices from all the

identified studies are combined into a single matrix, known as the “system matrix” (S) representing all the matrices $\{A^{(k)}, k = 1..m\}$. Each element of the system matrix S (s_{ij}) is defined as

$$S = \left\{ s_{ij} = \sum_{k=1}^m a_{ij}^{(k)} ; i = 1..N, j = 1 \dots N \right\} \quad [3]$$

where m is the total number of identified studies. Each element of the matrix S is normalised between -1 to 1 by dividing each element by the absolute maximum across all elements (Giles and others 2007)

$$S' = s'_{ij} = \frac{s_{ij}}{\max(|s_{ij}|)} \quad [4]$$

where $|s_{ij}|$ denotes the absolute value of s_{ij} . s'_{ij} gives the relative “weight of evidence” for the association between any nodes i and j . For simplicity, we will refer to S' as also the system matrix.

A.3 MEASURING THE STRUCTURAL PROPERTIES OF THE SYSTEM MATRIX

The structural properties of the system matrix (S') can be analysed quantitatively. Indices are calculated using graph theory. A centrality index shows how “well connected” a node i is in relation to other nodes. The centrality index (c_{ij}) is simply calculated by the sum of the total input connection values ($K_i^{(in)}$) to node j and the total output connection values ($K_j^{(out)}$) from node i

$$c_{ij} = K_j^{(out)} + K_i^{(in)} \quad [5]$$

where $K_i^{(in)}$ and $K_j^{(out)}$ are given respectively by

$$K_i^{(in)} = \sum_{j=1}^N |s'_{ji}| \quad [6]$$

$$K_j^{(out)} = \sum_{i=1}^N |s'_{ij}| \quad [7]$$

A.4 SIMULATING CAUSAL PROCESSES

To explore causal interactions and processes (including feedbacks), a dynamic simulation is conducted as follows. Denote by ($V^{(t)}$) the n -dimensional state vector of the system at iteration t . Each n^{th} element of the state vector represents the state of “causal activity” of the n^{th} node. An input state vector (unit vector as initial condition) ($V^{(t)}$) at iteration t is multiplied by the system matrix S' (s'_{ij}) to generate a new vector ($V^{(t+1)}$) at iteration $t + 1$. The resulting vector ($V_j^{(t+1)}$) is repetitively multiplied by matrix S' until the state vector reaches a stable equilibrium level (Giles and others 2007; Özsesmi and Özsesmi 2004):

$$V_j^{(t+1)} = f \left(\sum_{i=1, j \neq 1}^N V_i^{(t)} \times (S'_{ij}) \right) \quad [8]$$

At each iteration each element (u) of the vector $V_j^{(t+1)}$ is normalised to be within the interval of $[0, 1]$ by applying pointwise a threshold function $\{f(u)\}$ (i.e. to each of its elements) (Bueno and Salmeron 2009; Özesmi and Özesmi 2004). The threshold function is a logistic continuous function which determines the degree of activation level of the nodes after every iteration until equilibrium is reached:

$$f(u) = \frac{1}{1 + e^{-u}} \quad [9]$$

The equilibrium state describes the steady-state stable causal configuration of the system. Each n^{th} value of the state vector represents the level of activation (“causal activity”) in the n^{th} node. The level of activation reflects how each node influences each other over a number of iterations. The level of activation is a value between 0 to 1, where 1 is the highest level of causal activity and 0 is the lowest level of causal activity. The purpose of the simulation process is to measure the steady-state activity in each node in terms of feedbacks and causal interactions.

Appendix B

The appendix provides a walk-through hypothetical example to guide the reader through the various steps of the approach described in the paper. The hypothetical example concerns the association between indoor temperature (cold), cardio-respiratory and psychosocial (wellbeing) conditions. The example is demonstrated in five pseudo-algorithmic steps.

Step #1 (Figure B.1): Combine separate casual diagrams into one system diagram. The figure shows schematically two separate studies (out of k studies) concerned with the associations of three variables (nodes), N_1 to N_3 . A diagram is constructed based on each study to show the causal links between the nodes. A connection matrix is formed for each diagram and then a system matrix is constructed by combining the matrices as shown schematically below (refer also to the section “Representing mathematically the combined causal diagram into a system matrix” in Appendix A, for the mathematical details).

INDIVIDUAL CAUSAL DIAGRAMS AND SYSTEM DIAGRAM

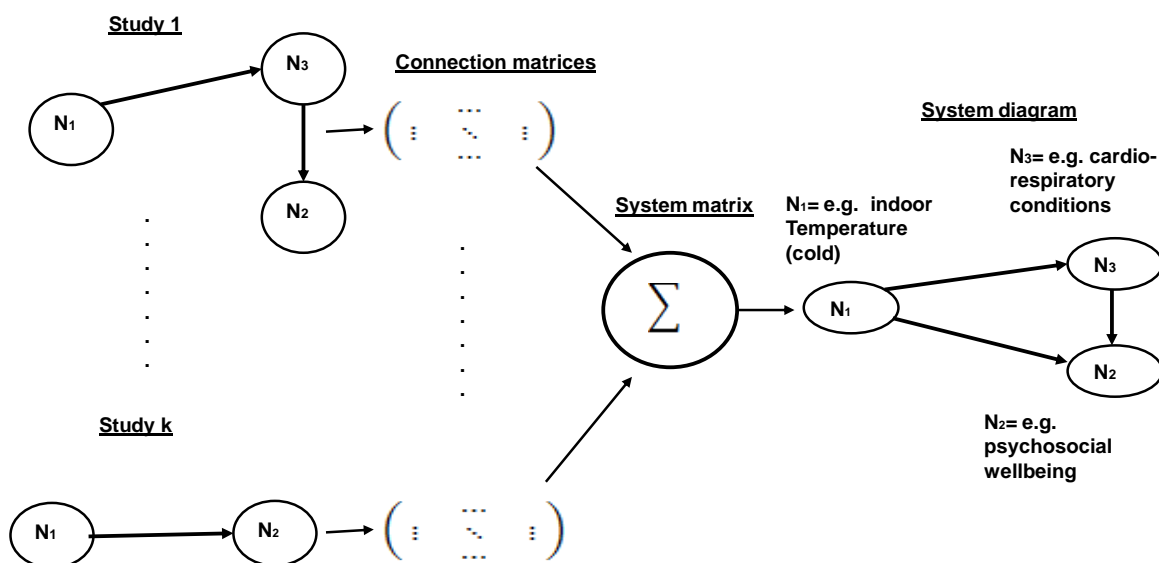
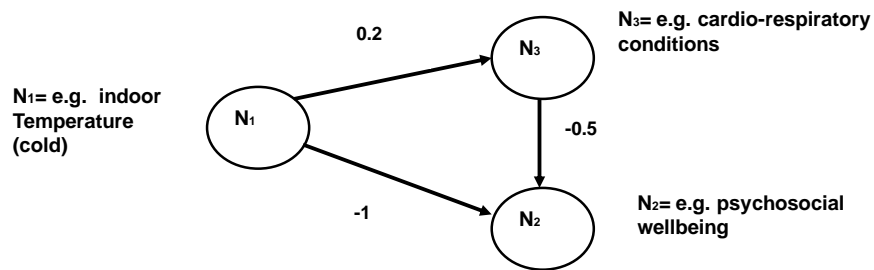


Figure B.1. Causal diagrams are constructed based on each study identified in the literature review. All causal diagrams are combined to form a “system diagram.”

Step # 2 (Figure B.2): Parameterise the system diagram. The figure shows the system diagram obtained by combining all the studies from the literature review reporting associations between the system variables. Causal weights between the nodes of a diagram in each study are obtained from reported measures of effects (e.g. odds ratio, percentage change, etc.). The causal weights in the system diagram represent the combined “strengths” or “relative weights” of evidence across all reported associations (see also section “Representing mathematically the combined causal diagram into a system matrix” and equation [4] in Appendix A).

It is important to note that the causal weights are algebraic quantities. A positive causal link (between nodes i and j) means when node i increases, node j increases. A negative causal link (between the same nodes i and j) on the other hand means that when node i increases, node j decreases. The amplitudes of the causal weights in a system diagram can be interpreted qualitatively as indicating the qualitative strength of the association e.g. “Strong” if the amplitude of the causal weight ≥ 0.90 ; “Medium” if it is ≥ 0.50 , and “Poor” if it is ≤ 0.50 .

THE SYSTEM DIAGRAM



Causal weight in the system diagram = s'_{ij} (e.g. $s'_{1,2} = -1$) derived and combined from all reported associations obtained in the literature review

Positive causality	Negative causality
Strong = 1	Strong = -1
Medium = 0.5	Medium = -0.5
Poor = 0.2	Poor = -0.2

Figure B.2. The System diagram represents the combination of all studies reporting associations from the literature review. Causal weights, assigned to each link in the system diagram represent the “strength” or “relative weight” of evidence. Causal weights can also be described qualitatively (as shown in the table).

Step # 3 (Figure B.3): Construct the system matrix from the system diagram .A system diagram (showing the causal links between nodes) can be represented by a matrix. For a system diagram with three nodes (N_1 to N_3), we construct a 3×3 system matrix (S') as follows. If there is a causal link associating node N_1 and N_3 , place the causal weight value as an entry in row N_1 and column N_3 in the matrix S' e.g. $s'_{1,3} = 0.2$. If a causal link is not directed between a pair of nodes, set the causal weight to 0 e.g. $s'_{2,3} = 0$ (refer to section “Constructing the causal diagrams from individual studies identified from the literature” in Appendix A for the mathematical details).

MATHEMATICAL MATRIX REPRESENTATION

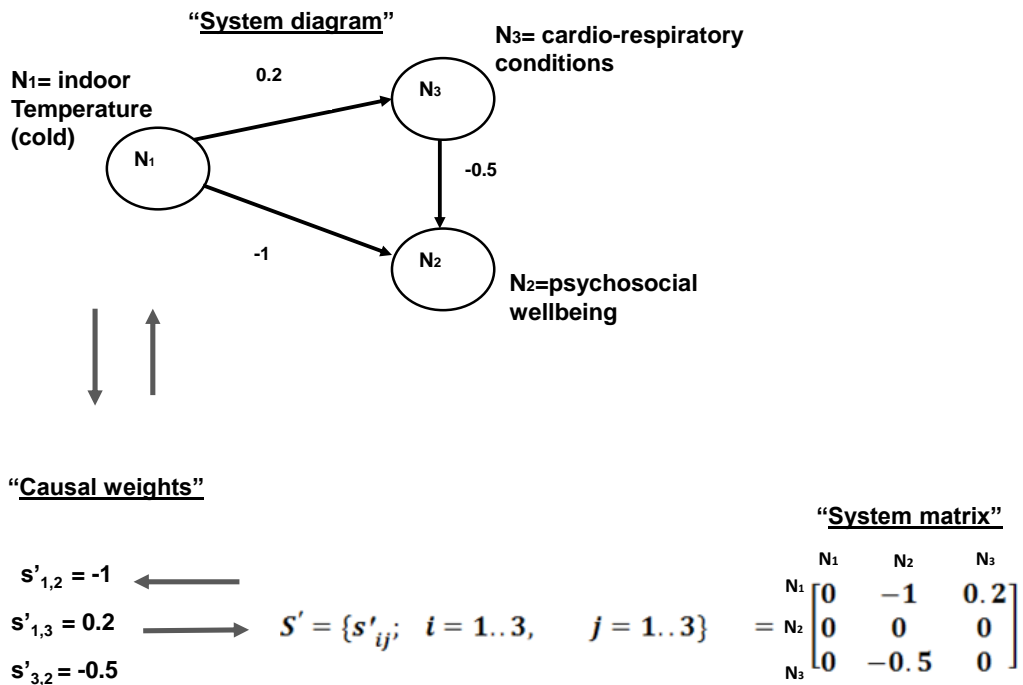
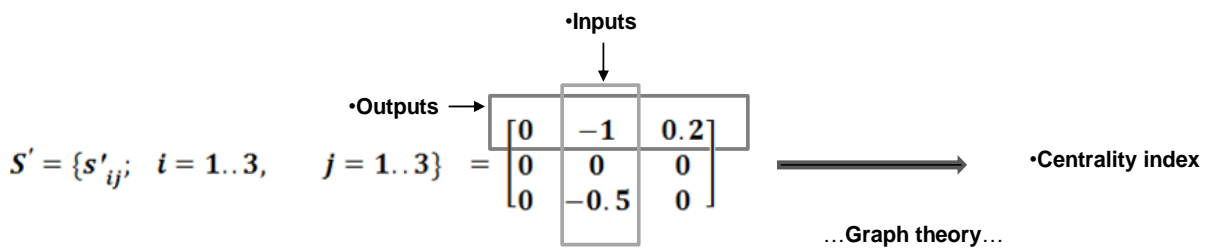


Figure B.3. The graphical structure of the system diagram can be represented mathematically in the form of matrix. Each causal weight value (positive or negative) is placed as an entry in the appropriate row and column in the matrix.

Step # 4 (Figure B.4): Determine the structural properties of the system using a quantitative measure called the centrality index. The centrality index is obtained by summing the total “inputs” and the total “outputs” connection values (causal weights) of the matrix (See equations [5], [6] and [7] in Appendix A). Input connection values are the values of the causal weights corresponding to the columns of the system matrix; output connection values are the values of the causal weights corresponding to the rows of the system matrix. The values of the centrality indices of the nodes and the outputs from the nodes and inputs to the nodes are shown in the table and figure below.

CENTRALITY INDEX



e.g. Distribution of Centrality Values

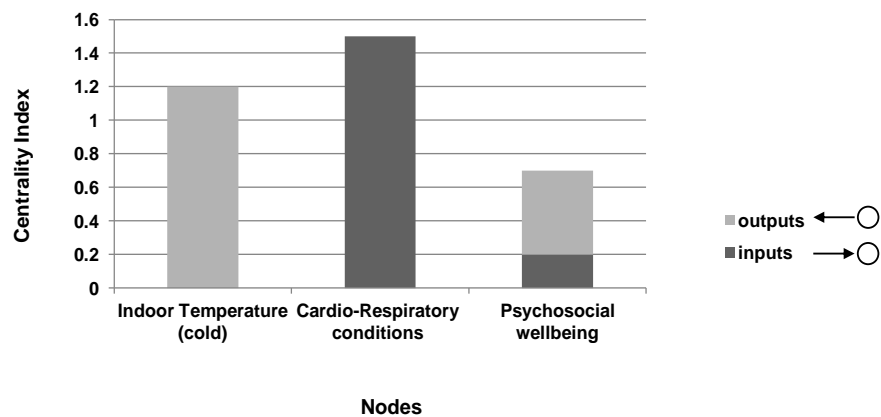


Figure B.4. The structural properties of the system can be analysed once the system matrix is obtained using the centrality index. The centrality index is calculated by the total sum of the absolute values of column and row entries in the system matrix.

Table B.4 Summary of centrality index, inputs and outputs

Nodes	inputs	outputs	Centrality
N1= Indoor Temperature (cold)	0.00	1.20	1.20
N2= Cardio-Respiratory Conditions	1.50	0.00	1.50
N3= Psychosocial wellbeing	0.20	0.50	0.70

Step # 5 (Figure B.5): Simulate the causal processes. The simulation of the causal processes is guided through an iterative calculation procedure. The overall result of the simulation is presented in a graph and in a table as shown below.

The dynamic simulation process is conducted in two iterative sub-steps which consist of: (1) matrix multiplications and (2) application of a threshold function. The two iterative sub-steps are explained as follows. An initial state vector (unit vector) is multiplied by a matrix $S'(s'_{ij}, i = 1..N, j = 1..)$ and then each element (u) of the resulting vector is normalised using a threshold function $\{f(u)\}$ to create a new normalised state vector ($V_j^{(t+1)}$). In subsequent iterations, the state vector ($V_j^{(t+1)}$) is repeatedly multiplied by the matrix S' until all state vector values reach an equilibrium (refer to the section “Simulating the Causal Process” and equations [8] and [9] in Appendix A, for the mathematical details).

COMPLEX CAUSAL PROCESS SIMULATION

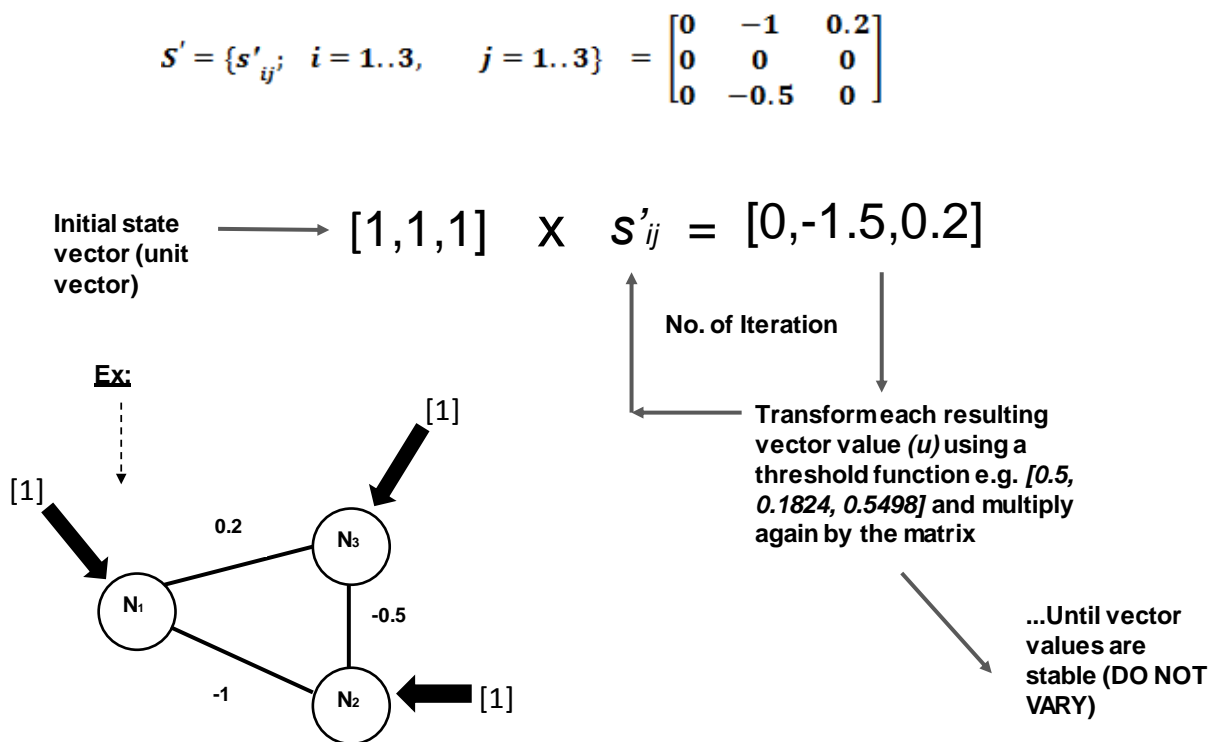


Figure B.5. Simulating causal processes consist in two iterative steps: (1) matrix multiplication by a state vector and (2) application of a threshold function.

The steps below give the details of the simulation calculations for the hypothetical example.

- 1) Matrix multiplication by a state vector:

$$V_j^{(t+1)} = f \left(\sum_{i=1, j \neq i}^N V_j^{(t)} \times (s'_{ij}) \right)$$

where $V = 1 \times n$ is a state vector that contains the values of the nodes.

$$S' = \{s'_{ij}; i = 1..3, j = 1..3\} = \begin{bmatrix} 0 & -1 & 0.2 \\ 0 & 0 & 0 \\ 0 & -0.5 & 0 \end{bmatrix}$$

Initial state vector (unit vector) $V = [1,1,1]$ represents weakly perturbed inputs assigned to each node.

$$\text{Resulting vector} = V \times s'_{ij} = [1,1,1] \times \begin{bmatrix} 0 & -1 & 0.2 \\ 0 & 0 & 0 \\ 0 & -0.5 & 0 \end{bmatrix} = [0, -1.5, 0.2] = [u1, u2, u3]$$

2) Application of threshold function:

$$f(u) = \frac{1}{1 + e^{-u}}$$

$$N_1 = f(u1) = \frac{1}{1 + e^{-1 \times 0}} = 0.5$$

$$N_2 = f(u2) = \frac{1}{1 + e^{-1 \times -1.5}} = 0.1824$$

$$N_3 = f(u3) = \frac{1}{1 + e^{-1 \times 0.2}} = 0.5498$$

New state vector $V^{(1)} = [0.5, 0.1824, 0.5498] = \text{Iteration No. 1}$

New state vector $V^{(2)} = [0.5, 0.556034, 0.524979] = \text{Iteration No. 2}$

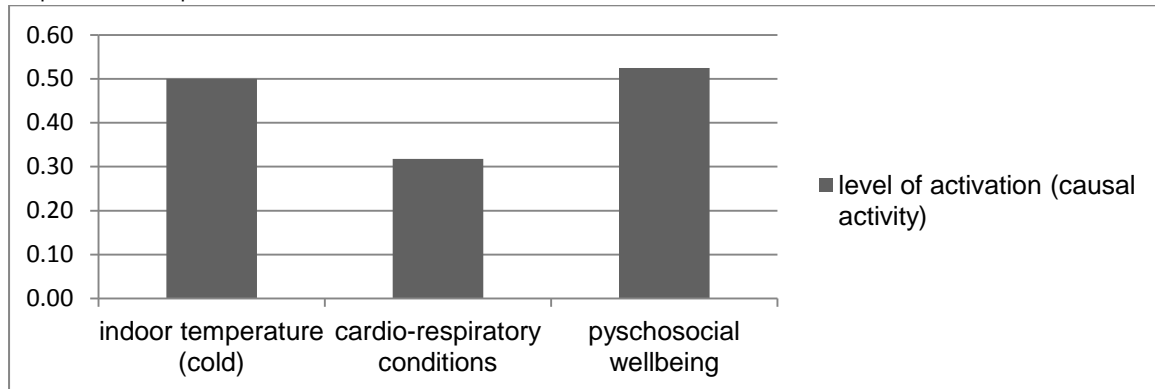
New state vector $V^{(3)} = [0.5, 0.559100, 0.524979] = \text{Iteration No. 3}$

Until stable pattern conditions (equilibrium) are reached

Table B.5 Summary of iterations

No. of iterations until stable conditions (equilibrium):	1	3	2
	Node 1	Node 2	Node 3
	1.000000	1.000000	1.000000
Iteration No.1	0.500000	0.182426	0.549834
Iteration No. 2	0.500000	0.315416	0.524979
Iteration No. 3	0.500000	0.318106	0.524979
Steady state	0.500000	0.318106	0.524979
	0.500000	0.318106	0.524979

Graph B.5 Causal processes between the nodes after simulation



The stable pattern reached at equilibrium describes the steady-state system behavior taking into account the feedback processes and the pattern of causal interactions between the three variables: indoor temperature (cold), cardio-respiratory and psychosocial (wellbeing). Each element of the steady-state state vector represents the equilibrium level causal activity (level of activation) in the individual node (variable). The level of activation measures how each node influence one another over a number of iterations. The result of the simulation process describes how the system functions based on its structural properties. To understand a causal system fully, it is important to analyse its structural properties as well as its dynamic causal behavior.

References

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